uncontrolled modes and not use them in the controller feedback, however, this requires more real-time computation power.

IV. Conclusions

Stabilization of flexible structures with naturally passive input/output pairs have been extensively studied in the past. For nonpassive pairs, direct negative feedback requires small gain and has severely limited gain and phase margins. This Note presents an observer-based extension of the passive controller design to the case of nonpassive input/output pairs. The selection of two design parameters: passive output map and observer gain are discussed based on the performance, robustness, and noise sensitivity considerations. Experimental results have demonstrated the basic viability of this method.

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Optimal Sensor Placement for Modal Identification Using System-Realization Methods

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Introduction

SENSOR placement is an important issue that must be addressed by engineers working problems in identification, analysis, control, and health monitoring of large flexible structures. Until recently, optimal sensor placement for modal identification has not been studied extensively because, for a typical modern ground vibration test, a large number of sensors is usually available to the test engineer. There are, however, currently important modal identification problems in which the number of available sensors is very limited and their positions are essentially fixed once in service.

This Note considers the Effective Independence (EfI) sensor placement method proposed by Kammer.[1] The EfI method maximizes both spatial independence and signal strength of the targeted finite element model mode shapes Φ_f, partitioned to the corresponding sensor locations Φ_{fs}, by maximizing the determinant of an associated Fisher information matrix given by $Q = \Phi_{fs}^T \Phi_{fs}$. It has been shown[2] that the EfI of the ith sensor E_{Di} is related to the determinant of the information matrix by the expression

$$E_{Di} = \frac{|Q| - |Q_{Ti}|}{|Q|} \tag{1}$$

in which Q_{Ti} represents the information matrix with the ith candidate sensor location deleted from the target modes. Therefore, E_{Di} represents the fractional change in the determinant of the information matrix if the ith candidate sensor location is deleted. The EfI process proceeds by sorting the entries in E_D and deleting the lowest ranked sensor. The remaining sensor locations are then reranked. In an iterative manner a large candidate set of sensor locations can be quickly reduced to the desired number. Although the EfI method has been shown to place sensors to the benefit of posttest correlation and model updating, it is not clear that the method enhances the modal identification process itself. The contribution of this Note is the presentation of the formal relationship between the EfI sensor placement technique and system-realization methods of modal identification. A currently popular system-realization method, called the eigensystem realization algorithm (ERA),[3] is considered.

Effects of EfI on the Observability Matrix

Realization methods for modal identification rely on a generalized observability matrix V_p. For the system

$$\begin{aligned} \dot{z} &= Az + Bu \\ y_s &= Cz + Nu \end{aligned} \tag{2}$$

the generalized observability matrix is given by

$$V_p = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix} \tag{3}$$

In general, most structural systems are not proportionally damped. Therefore, it is important to examine the effects of nonproportional damping on the sensor placement problem for modal identification. It is assumed that there are no rigid-body modes in the target mode set. In normal mode coordinates, the system matrix in Eqs. (2) is given by

$$A = \begin{bmatrix} 0 & I \\ -\omega^2 & -D_q \end{bmatrix} \tag{4}$$

in which ω is a diagonal matrix of modal frequencies. The normal modes decouple the physical mass and stiffness matrices, but not the damping matrix, D. The modal damping matrix D_q given by

$$D_q = \Phi_f^T D \Phi_f \tag{5}$$

is, in general, fully populated. To identify the k responding target modes, V_p must be full column rank, i.e., $\mathrm{rk}(V_p) = 2k$. In modal coordinates, V_p can be decomposed into the product of two matrices

$$V_p = S_p(\Phi_{fs}) Z_p(\omega, D_q) \tag{6}$$

where S is a $(pn_s \times pk)$ matrix function of Φ_{fs} and Z is a $(pk \times 2k)$ matrix function of ω and D_q. An optimum sensor configuration will minimize the required size of the generalized observability matrix while still maintaining its rank. The smallest possible observability matrix that still has rank $2k$ is given by

$$V_2 = \begin{bmatrix} C \\ CA \end{bmatrix} \tag{7}$$

In the case of accelerometers, the output influence matrix is given by

$$C = \begin{bmatrix} -\Phi_{fs}\omega^2 & -\Phi_{fs} D_q \end{bmatrix} \tag{8}$$

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which results in the minimum observability matrix

$$V_2 = \begin{bmatrix} -\Phi_{fs}\omega^2 & -\Phi_{fs}D_q \\ \Phi_{fs}D_q\omega^2 & -\Phi_{fs}\omega^2 + \Phi_{fs}D_q^2 \end{bmatrix} = S_2 Z_2 \qquad (9)$$

where S_2 and Z_2 are $(2n_s \times 2k)$ and $(2k \times 2k)$ matrices, respectively, given by

$$S_2 = \begin{bmatrix} \Phi_{fs} & 0 \\ 0 & \Phi_{fs} \end{bmatrix} \text{ and } Z_2 = \begin{bmatrix} -\omega^2 & -D_q \\ D_q\omega^2 & -\omega^2 + D_q^2 \end{bmatrix} \qquad (10)$$

According to Sylvester's inequality,[4] V_2 will have rank $2k$ if S_2 is full column rank and Z_2 is nonsingular. The determinant of Z_2 can be computed using the well-known expression

$$|Z_2| = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = |Z_{11}|\left|Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}\right| \qquad (11)$$

Substituting the partitions from Eq. (10) produces

$$|Z_{N2}| = |-\omega^2|\left|-\omega^2 + D_q^2 - D_q\omega^2(-\omega^2)^{-1}(-D_q)\right| = |\omega^4| \qquad (12)$$

which indicates that Z_2 is nonsingular regardless of the damping type and sensor placement. It is obvious from Eq. (10) that spatial matrix S_2 will be full column rank and, thus, the system will be observable if and only if the target mode shape partitions Φ_{fs} are full column rank. Therefore, at least k sensors must be placed such that the target mode partitions are linearly independent. This is precisely the objective of the EfI sensor placement methodology described earlier. The EfI method results in a minimization of the size of the generalized observability matrix required to produce a rank of $2k$.

The optimum sensor configuration should not only render the system observable, but it should also enhance the degree of observability. An observability grammian can be generated in the form

$$W_0 = V_2^T V_2 = Z_2^T S_2^T S_2 Z_2 \qquad (13)$$

Associated with the observability grammian is a hyperellipsoid in observability space given by

$$x^T W_0^{-1} x = 1 \qquad (14)$$

The volume of this hyperellipsoid is proportional to $|W_0|^{1/2}$, therefore, the determinant of the observability grammian is often used as a measure of the degree of observability of a system. Sensors should be placed such that the determinant of the observability grammian is maximized. Using Eq. (13), the determinant can be written as

$$|W_0| = \left|Z_2^T\right|\left|S_2^T S_2\right||Z_2| = |Z_2|^2\left|S_2^T S_2\right| \qquad (15)$$

It has already been shown that sensor placement has no effect upon the matrix Z_2 and thus also its determinant. On the other hand, the determinant of the matrix product $S_2^T S_2$ is given by

$$\left|S_2^T S_2\right| = \left|\Phi_{fs}^T \Phi_{fs}\right|^2 = |Q|^2 \qquad (16)$$

Maximization of the determinant of the information matrix thus results in the maximization of the degree of system observability. Thus, the EfI sensor placement technique also maximizes the observability of the system. Analogous results are obtained for displacement and velocity sensors. The important thing to note is that for sensor placement, the information matrix can be based solely on the normal modes of the system, regardless of the type of damping.

Effects of EfI on the Eigensystem Realization Algorithm

Considering a discrete time system, the ERA method is based upon a block Hankel data matrix that can be written as

$$H_{pd}(j) = V_p A_D^j W_d \qquad (17)$$

in which j represents the jth time step, W_d is a generalized controllability matrix given by

$$W_d = \begin{bmatrix} B_D & A_D B_D & \dots & A_D^{d-1} B_D \end{bmatrix} \qquad (18)$$

and A_D and B_D are the discrete versions of the system and input influence matrices in Eq. (2). It is assumed that the actuator locations are fixed such that all of the responding target modes are controllable, implying that the rank of W_d is $2k$.

The ERA method provides a minimum realization of the system that relies on the singular-value decomposition of the block Hankel matrix evaluated at $j = 0$, which for the minimum generalized observability matrix is given by

$$H_{2d}(0) = V_2 W_d = U O Y^T \qquad (19)$$

where U and V are orthonormal matrices and O is the corresponding matrix of singular values. To identify the k target modes, the block Hankel matrix must have rank $2k$. In general, the test data will contain noise, therefore, the rank of $H_{2d}(0)$ will be larger than $2k$ and there will be more than $2k$ positive singular values. To distinguish between target mode singular values and noise or computational singular values, the objective of the sensor placement method should be to maximize the magnitudes of the singular values of $H_{2d}(0)$ that correspond to the structural target modes. The minimum realization is generated by partitioning the matrix of singular values to the corresponding submatrix O_r. The associated minimum rank block Hankel matrix is then given by

$$H_{2d}(0) = V_2 W_d = U_r O_r Y_r^T \qquad (20)$$

in which U_r and Y_r represent the first $2k$ columns of U and Y, respectively.

The effect of sensor placement on the singular values of H_{2d} can be investigated by postmultiplying Eq. (20) by Y_r and taking advantage of the orthonormality of its columns, yielding

$$U_r O_r = V_2 W_d Y_r \qquad (21)$$

Premultiplying each side of Eq. (21) by its transpose produces

$$O_r^T U_r^T U_r O_r = Y_r^T W_d^T V_2^T V_2 W_d Y_r \qquad (22)$$

which, when utilizing the orthonormality of the columns of U_r, the diagonal form of O_r, and the definition of the observability grammian, becomes

$$O_r^2 = Y_r^T W_d^T W_0 W_d Y_r \qquad (23)$$

Noting that $W_d Y_r$ is a $(2k \times 2k)$ full rank matrix, the determinant of each side of Eq. (23) can be taken to produce

$$|Q_r|^2 = \left|Y_r^T W_d^T\right||W_0||W_d Y_r| = |W_0|\left|W_d Y_r Y_r^T W_d^T\right| \qquad (24)$$

The columns of Y_r span the row space of the generalized controllability matrix W_d, therefore, $Y_r Y_r^T$ is an orthogonal projector onto the column space of W_d^T. Using the fact that $Y_r Y_r^T W_d^T = W_d^T$, Eq. (24) can be expressed as

$$|O_r|^2 = |W_0|\left|W_d W_d^T\right| = |W_0||W_c| \qquad (25)$$

The geometric mean of the Hankel matrix singular values can then be expressed as

$$|O_r|^{1/2k} = |W_0|^{1/4k}|W_c|^{1/4k} \qquad (26)$$

Therefore, for fixed actuator locations, maximizing the determinant of the observability grammian maximizes the geometric mean of the Hankel matrix singular values. Thus, the EfI sensor placement approach enhances the separation of target mode singular values from computational singular values.

As shown by Peterson and Bullock,[5] the ERA method uses a least squares approach to estimate the discrete time system matrix A_D from the time shifted block Hankel matrix

$$H_{2d}(1) = V_2 A_D W_d \qquad (27)$$

The system matrix can then be estimated as

$$\hat{A}_D = V_2^+ H(1) W_d^+ \tag{28}$$

where the generalized inverses V_2^+ and W_D^+ are computed using the singular-value decomposition of the Hankel matrix $H(0)$ in which

$$V_2 = U_r O_r^{\frac{1}{2}} \qquad W_d = O_r^{\frac{1}{2}} Y_r^T \tag{29}$$

To investigate the effect of sensor placement, the least squares solution in Eq. (28) will be studied as a two-step process. In the first step, assume that the generalized inverse of W_d has been generated and used to postmultiply Eq. (27) to form the expression

$$H(1) W_d^+ = V_2 A_D \tag{30}$$

For fixed actuator locations, sensor placement has no impact upon the generalized inverse of the controllability matrix. Equation (30) represents a straightforward least squares estimation problem in which the discrete system matrix can be estimated as

$$\hat{A}_D = \left(V_2^T V_2\right)^{-1} V_2^T H(1) W_d^+ = (W_0)^{-1} V_2^T H(1) W_d^+ \tag{31}$$

where the Moore–Penrose form of the generalized inverse has been used to invert V_2 to illustrate the effect of sensor placement. For this estimation problem, the observability grammian is the corresponding Fisher information matrix. The best estimate for A_D is obtained by maximizing the determinant of the observability grammian.[6] Therefore, the EfI sensor placement methodology also provides a better estimate of the discrete time system matrix.

Conclusion

In cases where limited numbers of sensors are available for modal identification, it is vital that a systematic method is used for selecting their locations. A method called EfI has been previously devised to place sensors such that the locations maintain the linear independence of the modal partitions. This is required for posttest correlation and finite element model updating. This Note has shown analytically that the EfI method of sensor placement also enhances the modal identification process itself using system realization methods. It was shown that maximizing the independence of the target modes minimizes the size of the observability matrix required to render the system observable. This also minimizes the size of the required block Hankel data matrix used in the modal identification process. By maximizing the determinant of the Fisher information matrix, the EfI method maximizes the observability of the system and maximizes the singular values of the data matrix used in the ERA method to differentiate between target modes and computational modes. It also enhances the estimation of the discrete system matrix. It is believed that the EfI method provides an efficient technique for selecting a sensor configuration that not only provides the modal data required by structural dynamicists, but also produces an enhanced modal identification.

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Sliding Control Using Output Feedback

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Introduction

SLIDING mode control is a powerful robust control technique. It is well known for its design simplicity and robustness to parameter variations. One main drawback of the method is that system states are needed for implementation. There have been many papers on the issue of eliminating this drawback.[1–5] They can be grouped into two categories: static output feedback[1–3] and dynamic output feedback (observers).[4,5] Recently, Wang and Fan[1] proposed a new static output feedback sliding control method that incorporates the initial conditions of the outputs in the sliding variables. To implement their method, one would need to have additional logic for practical application In particular, in the case of regulation or tracking problems, mismatched disturbance could easily knock the trajectory off the desired path, and the control should be able to bring it back. Using Wang and Fan's method would require reinitializing the control after each such occurrence. This could be done by using additional logic. Such an ad hoc fix does not seem desirable.

Thus, it is more appropriate to devise a controller that provides robustness to all initial conditions, parameter variations and mismatched disturbance. The method in the next section can fulfill these requirements.

New Method

Consider a linear system of the form

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + d(t) \tag{1a}$$

$$\dot{x}_2 = (A_{21} + \Delta A_{21})x_1 + (A_{22} + \Delta A_{22})x_2 + B_2 u \tag{1b}$$

$$y = C_1 x_1 + C_2 x_2$$

with $x_1 \in R^{n-m}$, $x_2 \in R^m$, $x = [x_1^T \; x_2^T]^T$, $u \in R^m$, and $y \in R^p$. A_{ij}, B_2, C_1, and C_2 are matrices with appropriate dimensions. Here $d \in R^{n-m}$ denotes mismatched disturbance in the system. Assume the uncertainties ΔA_{21} and ΔA_{22} are all bounded and satisfy the so-called matching conditions, i.e.,

$$\Delta A_{21} = B_2 D_1 \tag{2a}$$

$$\Delta A_{22} = B_2 D_2 \tag{2b}$$

with the D_i unknown but bounded quantities. The sliding variable is defined as

$$S = Gy = GC_1 x_1 + GC_2 x_2 \tag{3}$$

with $S \in R^m$. G can be selected to satisfy Lemma 1 of Wang and Fan,[1] i.e.,

$$\operatorname{rank}[C_2 K - I] \le p - m \tag{4}$$

$$K = (GC_2)^{-1} G \tag{5}$$

with GC_2 of full rank. From Eq. (3), we can express x_2 in terms of S and x_1 to get

$$x_2 = (GC_2)^{-1}(-GC_1 x_1 + S) \tag{6}$$

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